COMS21202 – Symbols, Patterns and Signals

Problem Sheet B Solutions: Representations and Features

1 – Calculate the result of the convolution *A*B* in each of the examples below by hand.

(i) $A = \begin{pmatrix}1 & 2 & 1\end{pmatrix}$ $\quad B = \begin{pmatrix}2 & 2 & 3 & 3 & 2\end{pmatrix}$

(ii) $A = \begin{pmatrix}1 & 1 & 3 & 1 & 1\end{pmatrix}$ $\quad B = \begin{pmatrix}3 & 3 & -2 & -1 & 0 & 1 & 2 & 3 & 3\end{pmatrix}$

(iii) $A = \begin{pmatrix}-1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1\end{pmatrix}$ $\quad B = \begin{pmatrix}0 & 5 & 5 & 5 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 10 & 10 & 10 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 5 & 5 & 5 & 0\end{pmatrix}$

Now verify your result using the *conv* family of functions in Matlab. Use *help conv* to determine what convention Matlab uses when convolving at the border points.

Answer:

H is the result by hand using the convention seen in the lecture. ***M*** is the result using Matlab and the convention used by *conv* (which does not normalise and leaves it to the user).

(i) *Without normalisation*

$H = \begin{pmatrix}9 & 11 & 11\end{pmatrix}$ $\qquad M = \begin{pmatrix}2 & 6 & 9 & 11 & 11 & 7 & 2\end{pmatrix}$

With normalisation

$H = \frac{1}{4}\begin{pmatrix}9 & 11 & 11\end{pmatrix}$ $\qquad M = \begin{pmatrix}2 & 6 & 9 & 11 & 11 & 7 & 2\end{pmatrix}$

$M = M * 1/4$

(ii) Normalisation factor is $\frac{1}{7}$

$H = \begin{pmatrix}-1 & -1 & 0 & 7 & 13\end{pmatrix}$

$M = \begin{pmatrix}3 & 6 & 10 & 9 & -1 & -1 & 0 & 7 & 13 & 14 & 11 & 5 & 2\end{pmatrix}$

(iii) Normalisation factor is $\frac{1}{8}$

$H = \begin{pmatrix}-35 & 0 & 35 \\ -40 & 0 & 40 \\ -35 & 0 & 35\end{pmatrix}$ $\qquad M = \begin{pmatrix}0 & -5 & -5 & 0 & 5 & 5 & 0 \\ 0 & -15 & -20 & 0 & 20 & 15 & 0 \\ 0 & -25 & -35 & 0 & 35 & 25 & 0 \\ 0 & -30 & -40 & 0 & 40 & 30 & 0 \\ 0 & -25 & -35 & 0 & 35 & 25 & 0 \\ 0 & -15 & -20 & 0 & 20 & 15 & 0 \\ 0 & -5 & -5 & 0 & 5 & 5 & 0\end{pmatrix}$

2 – How would low pass filtering be achieved using the Fourier domain? In your answer describe what is meant by Cut-off Frequency.

Answer:

> Low pass filtering can be achieved by removing higher frequency information in the Fourier space, i.e. by retaining and letting lower frequencies pass through a filtering operation. Example filters are the ideal low pass filter and the Butterworth low pass filter. Some filter types have an abrupt cut-off point above which no higher frequencies are passed through, while others, like a Butterworth or Gaussian based filters, are more smoothly varying and do not have an abrupt cut-off point.

3 – Consider you are given the Fourier Transform space of an image. Using simple sketches to illustrate your answer, how would you select relevant regions to extract spectral features from

(a) only low frequency regions,
(b) only the very high frequency regions corresponding to prominent variations in intensity in the image that are at around 45 degrees to the horizontal,
(c) all approximately mid-range frequencies.

Answer:

Using conjugate symmetry, we can ignore the bottom half of the Fourier space and extract features from only the top half. We can then extract features from regions defined as

(a) for example a half
disc-shaped region with its centre at the centre of the Fourier space, i.e. at (u=0,v=0)

(b) a bar-shaped region with a small width, say 10, starting at around
(u= - max(ufreq.)/2, v= max(vfreq.)/2) angled at 135 degrees in the Fourier space

(c) a half-ring of a reasonable width, up to a maximum of 2*max(ufreq)/3, and starting from around (u=max(ufreq)/3, v=0).

NOTE: Exact u,v coordinates are not necessary, but approximations plus sketches should give the right indication, e.g. the half-disc must clearly be said to be at the centre of the space.

4 – Convolution in the spatial domain is equivalent to multiplication in the frequency domain, i.e. $f*g = FG$ (see Convolution lecture).

(a) Write a Matlab program that demonstrates $f*g = FG$ using any NxN image f and a $5x5$ averaging filter g consisting of all 1s. (Hint: you will need to pad g with zeros to make it the same size as the image before using fft2 in Matlab).
(b) Use Matlab's clock command to time how convolution in the spatial domain compares with multiplication in the Fourier domain.

Answer (Matlab):

(a) See file mc.m on the unit www page for the solution. The filter is created and then placed in the top-left part of a zero matrix of the same size as the image. This is the same effect as creating a filter and padding it with zeros till it is the same size as the image. The multiplication is then applied on an element-by-element basis of the two matrices.

(b) Use the *clock* and *etime* Matlab functions. First get the clock value before the main convolution or multiplication command. Then compute the elapsed time using the old clock value and a new clock reading AFTER the operation. Use *help etime* if unsure.

Answer (Python):

(a) See file mc.py on the unit www page for the solution.

(b) Use the *time()* Python function (form *time* library). First get the time value before the main convolution or multiplication command. Then compute the elapsed time using the old clock value and a new clock reading AFTER the operation, e.g.

```
import time
start = time.time()
...
end = time.time()
print(end - start)
```

5 – Imagine you have received a huge shipment of three variety of fruits consisting of *Oranges*, *Satsumas*, and *Red Pears*. The fruit is unfortunately mixed up, but you have access to a vision system you can program to distinguish and separate the fruit as they pass in front of a camera on a converyor belt one at a time. The camera is positioned to give a top-view of the fruit.

(a) State no fewer than three, and no more than five, features you would use in your design to distinguish between the different types. Very briefly explain why your features will pick the correct type each time considering that some measurements maybe somewhat affected by noisy data from the image acquisition process.

Answer:

Shape of fruit, colour of fruit, and size of fruit.
Shape is probably enough to locate pears (roundish for both oranges and satsumas, not round for pear) but due to noise, maybe colour could be used as an additional cue.
Size should be enough to locate satsumas, but best to be combined with colour for more certainty.
All three features should be combined to select oranges to deal with the possibly noisy measurements (i.e. larger than satsumas, rounder than pears, and more orangeness than pears).

(b) Consider you had actually been asked to consider using up to 20 features for this task. Discuss what would you do to find out which features are significant (or which ones are redundant)?

Answer:

Apply PCA to a training set of the data and keep only those features whose eigenvalues correspond to say 90% to 95% of the variance in the data. These should hopefully be substantially fewer than 20 dimensions.

6 – Use the “dopca” program available on the unit www page to upgrade it to a 3D case called “dopca3D” which does exactly the same tasks but on a 3D data set. You can make the data up if you like or use these values to add to the current 2D data in “dopca”:

```
[7.0 7.4 6.2 6.4 6.6 7.0 6.9 7.1 6.5 7.1];
```

Answer:

The first step is to add another set of data as a third dimension to matrix V. Continue by computing the mean and finding Vm3 and so on. Instead of *scatter* you will need to use *scatter3* and instead of *plot* use *plot3*, adding the appropriate extra parameters. You will also need to extend the array holding the principal components to 3D.
You should aim to see a 3D scatter view of the data and the three principal axes plotted in different colours.

An example of the way this extension can be implemented is linked to online. Use this only to nudge you along when you get stuck.

7 – Rotate an object, Fourier space rotates too. Translate an object, Fourier space translates too.

(a) Both statements are **True**.
(b) First statement is True and second one is False.
(c) First statement is **False** and the second one is **True**.
(d) Both statements are **False**.

8 – A 3x3 spatial filter with all elements set to **−1,** except the central element set to **16,** has a...

(a) normalisation factor of *1/8*
(b) normalisation factor of *1/16*
(c) normalisation factor of *1/24*
(d) normalisation factor of *1/12*

Sum the absolute value of each element, i.e. abs(-1)*8 + abs(16) = 24

9 – The eigenvalues of a dataset are: **[17, 11, 8, 2, 0.65, 0.35]**. What *variance* in the dataset do the first 3 eigenvalues represent?

(a) 93.2%
(b) 91.3%
(c) 96.3%
(d) 92.3%

(17+11+8) / (17+11+8+2+0.65+0.35) = 0.92307