# COMS21202 – Symbols, Patterns and Signals

## Problem Sheet: Representations and Transformations

1 – Using *sin(2πnx)*, demonstrate the concept of superposition as follows:

(a) first plot three sine functions over the range ±3 in steps of 0.1 using *n={1/4,1,2}*. Note, plots should appear in the same graph to give a better sense of what is happening.

(b) Now plot in a different colour the sum of all the sines above.

(c) Add more sine functions over the same range and repeat step (b).

**Answer (Matlab):**

(a) First define the range, say x = [-3:0.1:3]
The sine function plot over the specified range with *n=1/4* is then "plot(sin(2*pi*x*1/4))"
Hold the plot. Now plot again for the other values of *n.*

(b) Add the sines from (a) and plot the new function using 'r' as a parameter of the plot function to draw in red. See *help plot* if unsure of the syntax.

**Answer (Python):**

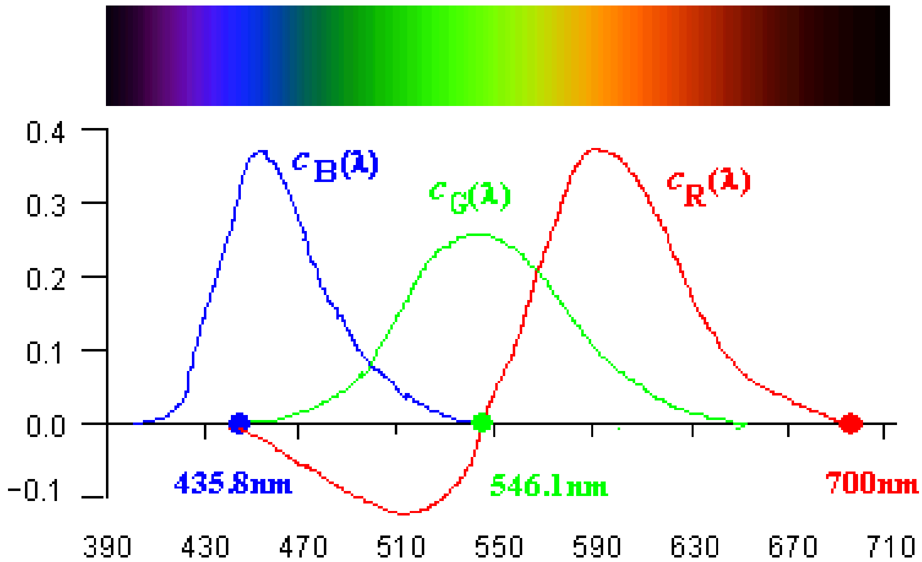
**(a)** Define the range, say: np.arange(-3, 3.1, 0.1)
The sine function plot over the specified range with n=1/4 is then
"plt.plot(x, np.sin(2*np.pi*x*1/4) )"
Now plot again for the other values of n on the same plot by using the same plot object.

(b) Add the sines from (a) and plot the new function using 'r' as a colour parameter of the plot function to draw in red. See help plt.plot if unsure of the syntax.

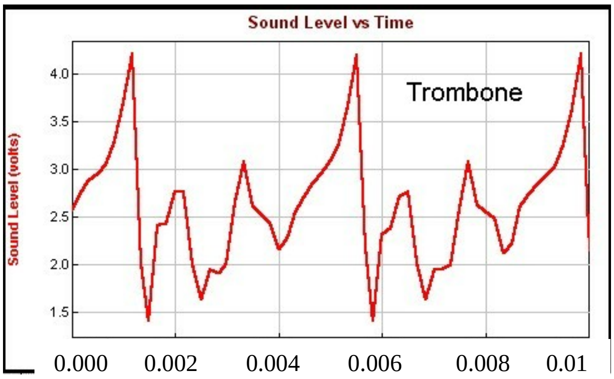
2 – What is White Light? Illustrate your answer with an approximate graph.

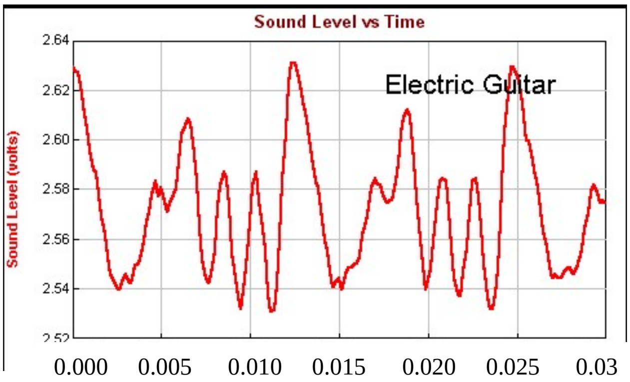
**Answer:**

White light is made up of a linear combination of variable wavelengths of each component colour (i.e. R,G, and B). Many (but not all) other colours can be induced by some linear combination of these three components, e.g. in digital terms, to get the colour *turquoise* you might mix 0.25*R+0.88*G+0.79*B. 'By superimposing all of them in equal amounts we get a spectral profile with energy distributed more or less uniformly over the whole visible spectrum, so it is perceived as white light.' Source: *http://www.mathpages.com/home/kmath578/kmath578.htm*



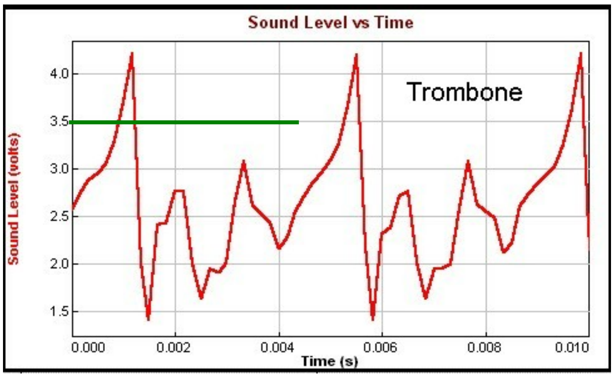

3 – The graphs below display the amplitude of the sound wave for a Trombone and an Electric Guitar as a function of time. The y-axis is the amplitude axis and the x-axis is the time axis. Notice that each one is plotted over a different length of time.

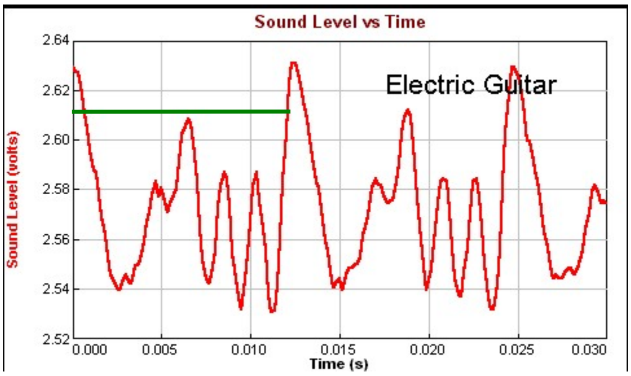



(a) Mark the period of the signal for each instrument.
(b) Approximately, how many periods are shown in these graphs for each instrument?
(c) Approximately, what is the peak amplitude in each case?
(d) Approximately, what is the frequency given the signal period in each case?
(e) Which signal contains higher frequency information? Why?

**Answer:**




(a) Marked in Green in the diagram above, about 0.0045 and 0.012 respectively.
(b) In both cases around 2 and a bit.
(c) Trombone: about 4.2 EG: about 2.63
(d) f = 1/T so 1/0.0045 = 222.2 and 1/0.012 = 83.3 respectively.
(e) The Trombone as it cycles more frequently than the EG over the same time period.

4- Determine which is an even and which is an odd function:

(i) $f(x) = 7x^3 - x$ odd

(ii) $f(x) = 3x^2 + 1$ even

(iii) $f(x) = 3x^2 \sin(x)$ odd

(iv) $f(x) = \dfrac{3}{(-x)^4 - 4}$ even

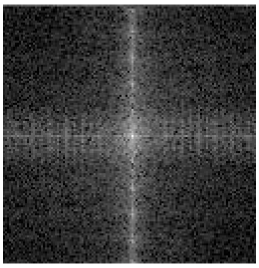
(v) $f(x) = \cos(x) + 5x - 3$ No symmetry of any kind, so it is neither even nor odd.

5 – Consider the two images (Sugar and Bricks) on the left. Identify which of the Fourier spaces (FS1 and FS2) on the right belongs to which image and explain clearly why.

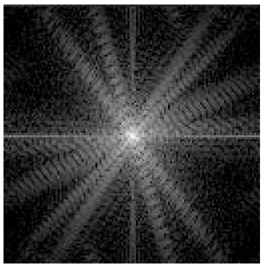
**Sugar**

**Bricks**

**FS1**

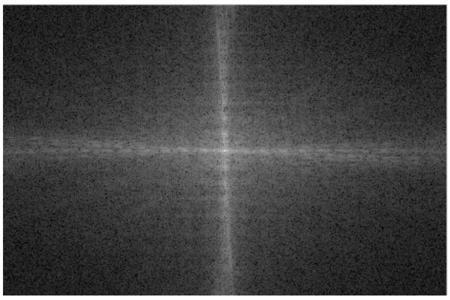


**FS2**

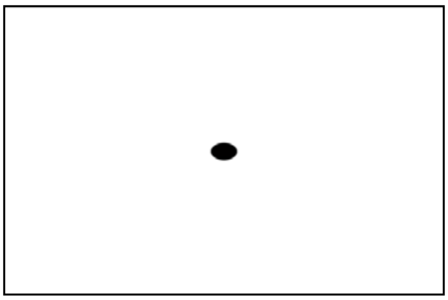


FS2 belongs to the sugar blocks image and FS1 belongs to the brick image. The high magnitude frequencies in FS1 are for the Brick image as they clearly signify the presence of very strong horizontal and vertical lines in that image. The angled lines in the sugar blocks image result in the strong non-horizontal and non-vertical directional lines in FS2.

6 – The figure below on the left shows an image of a building wall, with its Fourier Space magnitudes shown in the middle. A reconstructed image (inverse FFT image), after some manipulation of the Fourier magnitudes, is shown on the right. How should the Fourier space be manipulated (e.g., what kind of a mask could have been applied to it) to achieve this reconstructed result? Include a sketch to illustrate your answer.



Much of the contrast has been removed and an almost edge-map of the image has resulted. Edges signify high frequency changes in the image pixels. Hence, all this evidence points to a loss of low frequency magnitudes. The mask applied to the Fourier space magnitudes is therefore something similar to this:



7 – The following gene sequence contains significant frequencies. Design two different symbolic encodings and in each case apply your encoding to extract some of these frequencies.

**ACAGAGATACAGAGATACAG......**

*A=1, G=C=T=0* ➔ *10101010101010101… so period is 2, f=1/2*
*A=1,G=2,C=3,T=4* ➔ *1213131412131314121 3… so  period is 8, f=1/8*